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Condensation of spherical-cap shaped bubbles

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Abstract—In this investigation, an equation is theoretically developed to predict the collapse rate of spherical-cap shaped bubbles. Heat transfer at the top surface and in the wake of the bubble is determined. The comparison between the theoretical predictions and the experimental measurements of spherical-cap shaped steam bubbles condensing in subcooled water is satisfactory.

1. INTRODUCTION

AN EXTENSIVE amount of work has been carried out into the fluid mechanics and mass transfer of large bubbles, primarily in connection with underwater explosions, fluidized beds, and processing of liquid metals, and much of this has been the subject of a comprehensive review in a book on bubbles, drops and particles by Clift *et al.* [1]. But, there is a limited amount of experimental data and theoretical studies on condensation of spherical-cap shaped bubbles.

Danckwerts [2] introduced a surface renewal model for mass transfer at the wake of a spherical-cap bubble in the form

$$k_{\mathrm{m}} \sim Ds \tag{1}$$

where D is the diffusion coefficient and s the surface renewal rate. Lamont and Scott [3], assuming that mass transfer is controlled by small eddies, obtained a value of the average mass transfer coefficient as

$$k_{\mathrm{m}} \sim \left(\frac{D}{\nu}\right)^{0.5} (\varepsilon\nu)^{0.25} \tag{2}$$

where ε is the rate of energy dissipation by turbulence per unit mass. Coppus [4] estimated ε from the total energy dissipation of the bubble, Γ, as follows:

$$\Gamma = \mathrm{Drag} \cdot U = (\rho_{\mathrm{l}} g V_{\mathrm{b}}) U. \tag{3}$$

He assumed that all the energy is dissipated in the closed wake behind the bubble and determined the total energy dissipation rate per unit mass as

$$\varepsilon_{\mathrm{T}} = \frac{\Gamma}{\rho_{\mathrm{l}} V_{\mathrm{w}}} = \frac{g V_{\mathrm{b}} U}{V_{\mathrm{w}}} = \frac{gU}{V_{\mathrm{w}}/V_{\mathrm{b}}}. \tag{4}$$

He suggested that the energy dissipated by turbulence was equal to the total energy dissipation rate times a function of the Reynolds number or

$$\varepsilon = \varepsilon_{\mathrm{T}} f(Re) = \frac{gU}{V_{\mathrm{w}}/V_{\mathrm{b}}} f(Re) \tag{5}$$

where for a low Reynolds number (laminar wake flow) $f(Re) = 0$ and for a high Reynolds number (turbulent wake flow) $f(Re) = 1$.

2. THEORY

In some cases of the experimental data of steam bubbles condensing in subcooled water, it was observed that when a near spherical bubble detached from the orifice, it became slightly flattened at the rear of the bubble, passing through a hemispherical to a spherical-cap shape as illustrated in the collapse pattern shown in Fig. 1.

When a spherical-cap bubble rises, with a constant velocity U, through a subcooled liquid, heat transfer will take place from the top of the bubble and in addition some heat transfer will take place by conduction into the wake.

If we consider that the spherical-cap bubble shown in Fig. 2 has a constant radius R_0 and an angle γ, which decreases as collapse continues, we can, by assuming simplified potential flow over the spherical surface, determine the heat transfer from the top surface of the bubble.

The convection of heat in the water flowing round a spherical-cap bubble is assumed to be given by

$$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) \tag{6}$$

where conduction in the tangential direction is assumed to be much smaller than that in the radial direction. Assuming the thickness of the thermal boundary layer in the water flowing around the bubbles to be small, the velocity in the radial direc-

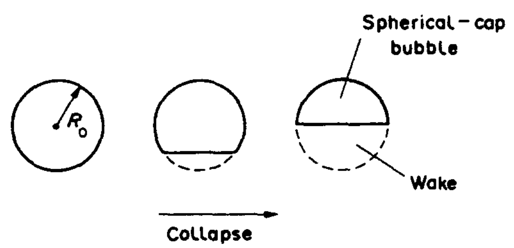

FIG. 1. Impressions of collapse pattern of spherical-cap bubbles.

NOMENCLATURE

A_r	surface area at the rear of the bubble [m^2]
C_1	variable defined by equation (25)
C_{bv}	constant defined by equation (23)
c_p	specific heat [J kg^{-1} K^{-1}]
D	diffusion coefficient [m^2 s^{-1}]
Fo	Fourier number, $\alpha_l t/4R_0^2$
g	gravitational acceleration [m s^{-2}]
h_{fg}	specific enthalpy of evaporation [J kg^{-1}]
Ja	Jakob number, $\rho_l c_p \Delta T/\rho_g h_{fg}$
k_m	mass transfer coefficient [m s^{-1}]
Pe	Peclet number, $2RU/\alpha$
Pe_0	Peclet number, $2R_0U/\alpha$
Q	rate of heat transfer from the bubble at time t [W]
q_θ	heat flux at angular position θ on the surface of the bubble [W m^{-2}]
R	bubble radius [m]
R_0	initial bubble radius [m]
$\dot{R}$	bubble wall radial velocity, $\mathrm{d}R/\mathrm{d}t$ [m s^{-1}]
Re	Reynolds number, $2RU/\nu = 2RU\rho/\mu$
Re_0	Reynolds number, $2R_0U/\nu$
r	radial coordinate [m]
s	surface renewal rate [s^{-1}]
T	temperature [K]
ΔT	subcooling of water relative to steam [K]
t	time [s]
U	bubble rise velocity [m s^{-1}]
u_r	velocity in the radial direction [m s^{-1}]
u_θ	velocity in the tangential direction [m s^{-1}]
y	radial distance from the bubble surface, $r-R$ [m]
Z	dimensionless bubble volume, $V/\frac{4}{3}\pi R_0^3$.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
β	dimensionless radius based on initial bubble radius, R/R_0
γ	polar angle from the vertical defining the base of the spherical-cap bubble [rad]
ε	rate of energy dissipation by turbulence per unit of mass [W kg^{-1} or m^2 s^{-3}]
ε_T	total rate of energy dissipation per unit of mass [W kg^{-1} or m^2 s^{-3}]
θ	polar angle from vertical [rad]
ν	kinematic viscosity, μ/ρ [m^2 s^{-1}]
ρ	density [kg m^{-3}].

Subscripts

b	bubble
f	fluid, front
l	liquid
m	mean
T, t	total
v	vapour
w	wake
0	initial value.

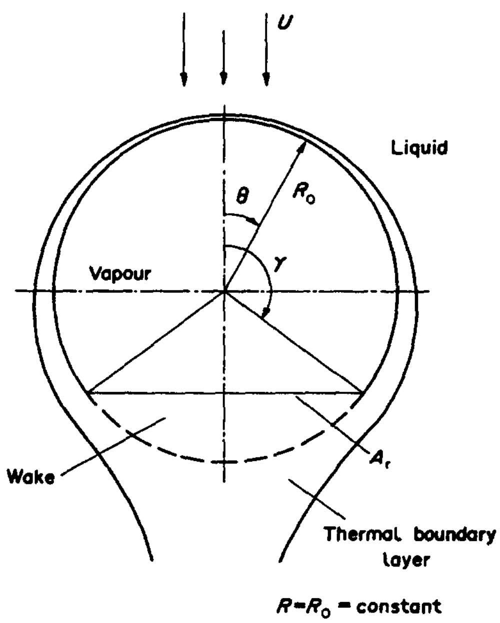

Fig. 2. Theoretical model for collapse of spherical-cap bubbles.

tion u_r and in the tangential direction u_θ in this layer are given by

$$u_r = -3U\frac{y}{R_0}\cos\theta + \dot{R}\left(1-2\frac{y}{R_0}\right) \tag{7}$$

$$u_\theta = \tfrac{3}{2}U\sin\theta \tag{8}$$

where y is the distance $(r-R)$ from the bubble surface. If we neglect bubble radial velocity (i.e. $\dot{R}=0$), the velocity u_r can be written as

$$u_r = -3U\frac{y}{R_0}\cos\theta. \tag{9}$$

For these conditions, Ruckenstein [5] gave the heat flux through the liquid boundary layer at angle θ as

$$q_\theta = c_p\rho_l\Delta T\frac{\frac{3}{2}UR_0\alpha\sin^2\theta}{\sqrt{\left(\frac{3}{2}\pi\alpha UR_0^3\left(\frac{2}{3}-\cos\theta+\frac{\cos^3\theta}{3}\right)\right)}}. \tag{10}$$

Thus, the heat transferred at the upper (front) surface, between $\theta = 0$ and γ, is given as

$$Q_{\gamma f} = \int_0^{\gamma} 2\pi R_0^2 \sin\theta \, q_\theta \, d\theta \tag{11}$$

$$= \frac{2\pi R_0^2 c_p \rho_l \Delta T \frac{3}{2} U R_0 \alpha}{\sqrt{(\frac{3}{2}\pi\alpha U R_0^3)}} \int_0^{\gamma} \frac{\sin^3\theta \, d\theta}{(\frac{2}{3}-\cos\theta+\frac{1}{3}\cos^3\theta)^{1/2}}. \tag{12}$$

Let

$$y = \tfrac{2}{3}-\cos\theta+\tfrac{1}{3}\cos^3\theta \tag{13}$$

$$\therefore \, dy = (\sin\theta-\cos^2\theta\sin\theta)\,d\theta = \sin^3\theta\, d\theta. \tag{14}$$

Hence

$$\int_0^{\gamma} \frac{\sin^3\theta\, d\theta}{(\frac{2}{3}-\cos\theta+\frac{1}{3}\cos^3\theta)^{1/2}} = 2(\tfrac{2}{3}-\cos\gamma+\tfrac{1}{3}\cos^3\gamma)^{1/2}. \tag{15}$$

Therefore

$$Q_{\gamma f} = 3\sqrt{\left(\frac{2\pi}{3}\right)}\frac{R_0^3 c_p \rho_l \Delta T U \alpha}{\sqrt{(\alpha U R_0^3)}} 2(\tfrac{2}{3}-\cos\gamma+\tfrac{1}{3}\cos^3\gamma)^{1/2} \tag{16}$$

$$\therefore \, Q_{\gamma f} = 6\sqrt{\left(\frac{\pi}{3}\right)}(c_p\rho_l\Delta T)\, Pe_0^{1/2}\, \alpha R_0(\tfrac{2}{3}-\cos\gamma + \tfrac{1}{3}\cos^3\gamma)^{1/2}. \tag{17}$$

In addition to this heat transferred at the top surface of the bubble, heat may also be conducted into the water in the wake, and this can be determined by using a method similar to that of Coppus [4].

Since we have turbulent flow, equation (5) becomes

$$\varepsilon = \frac{gU}{V_w/V_b}. \tag{18}$$

Substituting this in equation (2) and inserting a constant of proportionality of 0.10 gives

$$k_m = 0.10\left(\frac{D}{\nu}\right)^{0.5}\left(\frac{gU\nu}{V_w/V_b}\right)^{0.25} \tag{19}$$

or

$$\frac{k_m R}{D} = 0.10\left(\frac{\nu}{D}\right)^{0.5}\left(\frac{gR}{8U^2}\right)^{0.25} Re^{0.75}\left(\frac{V_b}{V_w}\right)^{0.25}. \tag{20}$$

By using an analogous heat transfer relationship which substitutes $h/\rho_l \cdot c_p$ for the mass transfer coefficient k_m and the thermal diffusivity α for the diffusion coefficient D, we can write

$$\frac{hR}{\rho_l c_p \alpha} = 0.10\left(\frac{\nu}{\alpha}\right)^{0.5}\left(\frac{gR}{8U^2}\right)^{0.25} Re^{0.75}\left(\frac{V_b}{V_w}\right)^{0.25} \tag{21}$$

$$\therefore \frac{hR}{\rho_l c_p \alpha} = C_b\, Pe^{0.50}\, Re^{0.25}\left(\frac{V_b}{V_w}\right)^{0.25} \tag{22}$$

where

$$C_b = 0.10\left(\frac{gR}{8U^2}\right)^{0.25}. \tag{23}$$

Assuming a closed spherical wake

$$\frac{V_b}{V_w} = \frac{\pi R^3\left(\frac{2}{3}-\cos\gamma+\frac{\cos^3\gamma}{3}\right)}{\frac{4}{3}\pi R^3 - \pi R^3\left(\frac{2}{3}-\cos\gamma+\frac{\cos^3\gamma}{3}\right)} = \frac{C_1}{\frac{4}{3}-C_1} \tag{24}$$

where

$$C_1 = \tfrac{2}{3}-\cos\gamma+\frac{\cos^3\gamma}{3}. \tag{25}$$

Hence, equation (22) can be written as

$$\frac{hR}{\rho_l c_p \alpha} = C_b\, Pe^{0.5}\, Re^{0.25}\left(\frac{C_1}{\frac{4}{3}-C_1}\right)^{0.25}. \tag{26}$$

Thus, heat transferred to the wake at the rear of the bubble is expressed as

$$Q_{\gamma r} = hA_r\Delta T = h\Delta T\pi(R\sin\gamma)^2 \tag{27}$$

where A_r is the surface area at the rear of the bubble.

$$\therefore \, Q_{\gamma r} = C_b\rho_l c_b \pi\Delta T R^2 \sin^2\gamma \times \frac{\alpha}{R}\, Pe^{0.5}\, Re^{0.25}\left(\frac{C_1}{\frac{4}{3}-C_1}\right)^{0.25}. \tag{28}$$

Since $R = R_0 =$ constant, $Pe \cong Pe_0$, $Re \cong Re_0$

$$Q_{\gamma r} = \pi C_b(c_p\rho_l\Delta T) \times R_0\alpha\, Pe_0^{0.5}\, Re_0^{0.25}\sin^2\gamma\left(\frac{C_1}{\frac{4}{3}-C_1}\right)^{0.25}. \tag{29}$$

Combining equations (17) and (29) the total heat transfer from the bubble is obtained

$$Q_\gamma = Q_{\gamma f}+Q_{\gamma r}$$

$$Q_\gamma = (c_p\rho_l\Delta T)\, Pe_0^{0.5}\, R_0\alpha\left(6\sqrt{\left(\frac{\pi}{3}\right)}C_1^{0.5} + \pi C_b\, Re_0^{0.25}\sin^2\gamma\frac{C^{0.25}}{(\frac{4}{3}-C_1)^{0.25}}\right). \tag{30}$$

An energy balance on the steam bubble states that the total heat transferred from the bubble at time t is given as

$$Q_\gamma = -h_{fg}\rho_v\frac{dV}{dt} \tag{31}$$

where dV/dt is the rate of change of volume of the bubble.

Thus, equating equations (30) and (31)

$$\frac{dV}{dFo} = -4Ja\,Pe_0^{0.5}\left(6\sqrt{\left(\frac{\pi}{3}\right)}R_0^3 C_1^{0.5} + \pi C_b\,Re_0^{0.25}\,R_0^3\sin^2\gamma\frac{C_1^{0.25}}{(\frac{4}{3}-C_1)^{0.25}}\right) \quad (32)$$

where

$$Fo = \frac{\alpha t}{4R_0^2}.$$

Let

$$Z = \frac{V}{V_0} = \frac{V}{\frac{4}{3}\pi R_0^3} = \frac{3}{4}\left(\frac{2}{3}-\cos\gamma+\frac{\cos^3\gamma}{3}\right) = \frac{3}{4}C_1 \quad (33)$$

$$\therefore dZ = \frac{1}{\frac{4}{3}\pi R_0^3}dV.$$

Hence, equation (32) can be written as

$$\frac{dZ}{dFo} = -4Ja\,Pe_0^{0.5}\left(6\sqrt{\left(\frac{\pi}{3}\right)}\frac{C_1^{0.5}}{\frac{4}{3}\pi} + \pi C_b\,Re_0^{0.25}\frac{1}{\frac{4}{3}\pi}\sin^2\gamma\frac{C_1^{0.25}}{(\frac{4}{3}-C_1)^{0.25}}\right) \quad (34)$$

$$\therefore \frac{dZ}{dFo} = -4Ja\,Pe_0^{0.5}\left(\frac{3Z^{0.5}}{\sqrt{\pi}} + (\tfrac{3}{4}C_b\,Re_0^{0.25})\sin^2\gamma\frac{Z^{0.25}}{(1-Z)^{0.25}}\right). \quad (35)$$

Integrating gives

$$Fo = -\frac{1}{4Ja\,Pe_0^{0.5}} \times \int_{Z_0}^{Z}\frac{dZ}{\frac{3}{\sqrt{\pi}}Z^{0.5}+(\frac{3}{4}C_b\,Re_0^{0.25})\frac{\sin^2\gamma\,Z^{0.25}}{(1-Z)^{0.25}}}. \quad (36)$$

From equation (33)

$$dZ = \tfrac{3}{4}\sin^3\gamma\,d\gamma. \quad (37)$$

Therefore, equation (36) can be written in terms of γ as

$$Fo = -\frac{3}{16Ja\,Pe_0^{0.5}} \times \int_{\pi}^{\gamma}\frac{\sin^3\gamma\,d\gamma}{\frac{3}{2}\frac{\sqrt{3}}{\sqrt{\pi}}C_1^{0.5}+(\frac{3}{4}C_b\,Re_0^{0.25})\left(\frac{3}{4}\right)^{0.25}\frac{\sin^2\gamma\,C_1^{0.25}}{(1-\frac{3}{4}C_1)^{0.25}}}. \quad (38)$$

From the definition of Z in equation (33), the dimensionless radius β can be calculated as

$$\beta = \left(\frac{V}{V_0}\right)^{1/3} = Z^{1/3} = \left(\frac{3}{4}\left(\frac{2}{3}-\cos\gamma+\frac{\cos^3\gamma}{3}\right)\right)^{1/3}. \quad (39)$$

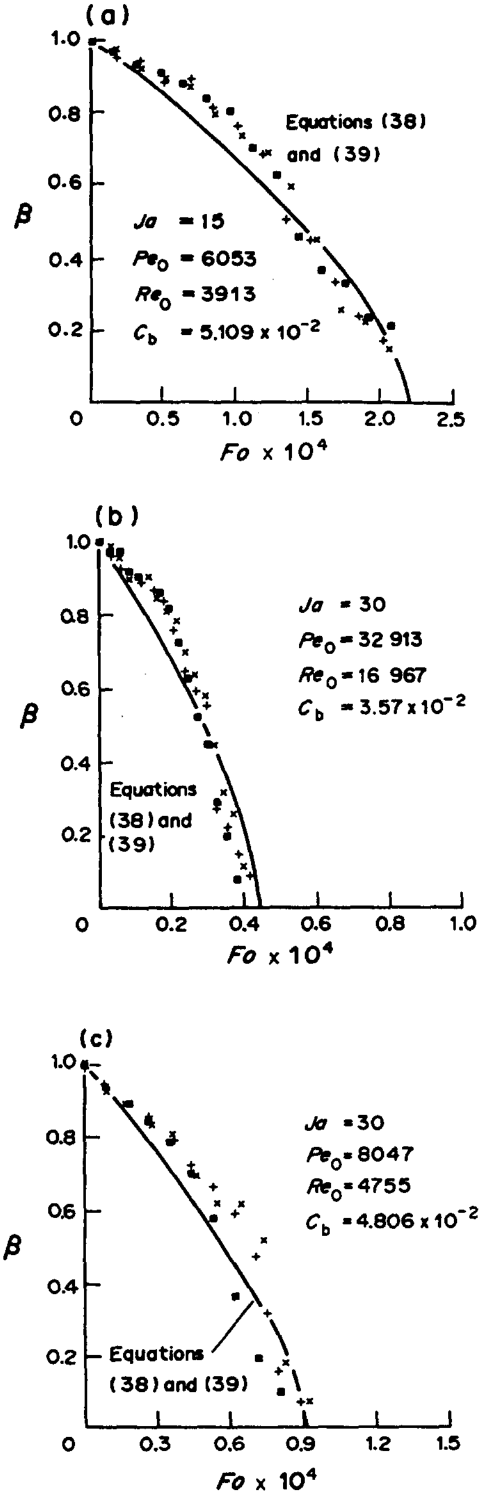

FIG. 3. Comparison of collapse data with the theory (+, × and ■ denote three different bubbles at the same conditions).

For a given experimental condition the Jakob number, Peclet number, Reynolds number and C_b are known. For a given angle, γ, equation (38) gives the Fourier number (Fo) and equation (39) determines the value of β.

Equation (35) can be rewritten as

$$\frac{\mathrm{d}Z}{\mathrm{d}Fo} = -4Ja\,Pe_0^{0.5}\left(\frac{3}{\sqrt{\pi}}Z^{0.5}+A\sin^2\gamma\left(\frac{Z}{1-Z}\right)^{0.25}\right) \tag{40}$$

where

$$A = \tfrac{3}{4}C_b\,Re_0^{0.25}.$$

When heat transfer at the rear of the bubble is neglected $A = 0$ and equation (40) reduces to

$$\frac{\mathrm{d}Z}{\mathrm{d}Fo} = -\frac{12}{\sqrt{\pi}}Ja\,Pe_0^{0.5}\,Z^{0.5}. \tag{41}$$

Hence

$$Z = \left(1-\frac{6}{\sqrt{\pi}}Ja\,Pe_0^{0.5}\,Fo\right)^2 \tag{42}$$

or

$$\beta = \left(1-\frac{6}{\sqrt{\pi}}Ja\,Pe_0^{0.5}\,Fo\right)^{2/3} \tag{43}$$

which is the equation given by Isenberg *et al.* [6] for the collapse of spherical bubbles rising freely with negligible bubble radial velocity.

3. COMPARISON OF THEORY WITH EXPERIMENTAL DATA

A simulation program was used to calculate and draw the β vs Fo curve according to equations (38) and (39). The theory is compared with our experimental data [7] where spherical-cap shaped steam bubbles condensing in subcooled water are chosen (Figs. 3(a)–(c)). Since initial bubble radius (R_0) and approximate bubble rise velocity (U) are known, Re_0, Pe_0 and C_b are calculated according to the definitions in the text. The theory agrees with the data reasonably well.

4. CONCLUSIONS

In the tests, especially at higher Jakob numbers, the collapse rate is lower during the early stages of collapse than the predicted collapse rate, but later on the rate of collapse increases so that the final collapse time shows reasonable agreement with that predicted, and in a few cases gives a faster collapse than the predicted rate. This collapse pattern may be due to the effect of the bubble condensation giving some heating of the water near the orifice and so reducing the effective driving force for condensation, while increased bubble distortion leads to a more rapid increase in the collapse rate towards the later stages of collapse.

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CONDENSATION DE BULLES A SOMMET SPHERIQUE

Résumé—Une équation est étabilie par voie théorique pour prédire le temps de collapsus des bulles dont la partie supérieure est sphérique. On détermine le transfert de chaleur à la surface supérieure et dans le sillage de la bulle. La comparaison des calculs et des mesures expérimentales de condensation des bulles de vapeur d'eau à sommet sphérique dans de l'eau sous-refroidie est satisfaisante.

KONDENSATION VON SCHIRMBLASEN

Zusammenfassung—Eine Gleichung zur Berechnung der Kondensationsgeschwindigkeit von Schirmblasen wird theoretisch entwickelt. Der Wärmeübergang an der Oberseite und im Nachlauf der Blase wird bestimmt. Die Übereinstimmung mit Versuchsergebnissen an Schirmblasen aus Wasserdampf, die in unterkühltem Wasser kondensieren, ist befriedigend.

КОНДЕНСАЦИЯ ПУЗЫРЬКОВ В ФОРМЕ СФЕРИЧЕСКИХ КОЛПАЧКОВ

Аннотация—Теоретически выводится уравнение для расчета скорости схлопывания пузырьков в форме сферических колпачков. Определен теплоперенос у верхней поверхности и в следе пузырька. Теоретические расчеты удовлетворительно согласуются с экспериментальными измерениями конденсирующихся в недогретой воде пузырьков пара, имеющих форму сферических колпачков.